

# Quantum Optics as a Relativistic Theory of Light

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## Abstract

*In analogy with Bohm's elucidation of special relativity, in this paper we criticize the Copenhagen interpretation and try to reinterpret quantum theory in the light of new evidence of quantum optics. In doing this, we are led to regard the real photon as only expressing an elemental relationship established between a quantized light field and a detector in the actual detection processes. With this kind of relational approach to quantum physics, new concepts are then needed to describe physical phenomena, and the mathematical structure of quantum theory of radiation is viewed as a conceptual map, in the same way as the Minkowski diagram in Einstein's special relativity, which already has the perspective of the observer implicit in it.*

**Key words:** relational approach to physics; basic physical laws as invariant relationships; inherent ambiguity in the meanings of physical objects, according to the Copenhagen interpretation; mathematical structure of quantum optics as a conceptual map; intrinsic unfalsifiability of Einstein-Podolsky-Rosen paradox

*We dedicate this essay to the memory of Professor David Joseph Bohm (1917-1992), whose brilliant exposition of the meaning of relativity we here convey.*

## 1. INTRODUCTION

Modern physics owes much to Galileo, who was the first to advocate experimentation and mathematical description of natural processes. In his opinion, any reasonable theory in physics should be based on measured facts of observation, for it might extricate, to some extent, description and deduction from ambiguity and groundless speculation. Along this line, Newton discovered classical mechanics, which explains the motion of man-sized objects; at the same time, he also provided more precise definitions to those absolute concepts of space, time, and physical objects.

A great transition in modern physics, however, was rendered by Einstein, who realized that *those observable properties (immediate facts) were only relational properties*. In other words, *by measurement we establish physical relationships between an "object" and the measuring apparatus*. This is in sharp contrast to Newtonian physics, in which an observation or measurement is that of the absolute property of the "object" itself. In this way, the objectivity of nature then will manifest itself as the possibility of finding what is invariant underlying those relational properties that are observable, and we shall consider those "objects," like space, time, and physical objects, not as absolute, but as relative invariants, which are abstracted from those relationships within certain observation domains. As the domain under investigation is broadened, we may expect to come to new invariant relationships containing the older ones as approximations and limiting cases.<sup>(1)</sup> Thus this bilateral trait of

physical observation has permanently cast the basic pattern of cognition by which man learns of the world. It is just as the principle of relativity goes: "The general laws of nature are to be expressed by relationships which are generally invariant, and which are in principle observable."

Early this century Einstein proposed that space and time coordinates are only relative properties consisting of relationships of objects and events to the measuring instruments and usually fail to be independent beings in broader domains. Instead, they fuse into one, as a four-dimensional continuum in which the existent exists. Thus Einstein radically reformed two concepts out of the three absolute notions provided by Newton.

Among these three basic notions the concept of physical objects is at the heart of our metaphysics and physics thinking. Objects with figure and size and motion under the law of causality are indeed more concrete and more direct than space and time. Newton wrote the following:

It seems probable to me, that God in the beginning form'd matter in solid, massy, hard, impenetrable, moveable particles, of such sizes and figures, and with such other properties, and in such proportion in space, as most conduced to the end for which he form'd them; and that these primitive particles being solids, are incomparably harder than any porous bodies compounded of them; even so hard, as never to wear or break in pieces; no ordinary power being able to divide what God himself made one in the first Creation.<sup>(2)</sup>

With this particle model of matter, not only did Newton succeed in describing the behavior of man-sized objects, but also

man could in later centuries overstep the bounds to exploit the atomistic structure that qualitatively accounted for a variety of natural phenomena ranging from electricity and magnetism to chemistry. Thus when this model came out at the end of the nineteenth century, it already seemed to be an “inevitable” truth, and many people then believed that the foundation of physics had been soundly established based on the mechanistic concept of nature.

As atom-sized phenomena were further investigated in the early days of this century, classical physics incurred several grave problems such as the “orbital collapse” of the Rutherford model. To save this planetary structure of the atom, Bohr had no choice but to introduce quantum rules. Those phenomenological rules, indeed, could explain the spectral data of simple atoms like hydrogen, yet they seemingly could not go on to quantitatively predict more complex systems, since formulas of this old quantum theory contained physical quantities (electronic orbits of definite dimensions and periods) lacking definition of measurement.<sup>(3)</sup> However, what concerned the physicists of this time more was how to construct a theory. In the eyes of positivism the theory based on quantities without measuring meaning was a kind of metaphysics.

Influenced by the idea of employing only observable quantities in Einstein’s relativity theory, Heisenberg made a great step forward in 1925.<sup>(4)</sup> He successfully bypassed those unobservables by using other existing quantitative relationships of empirical facts to found quantum formalism effectively. For over seventy years since its establishment, this formalism has been crowned with amazing success in nearly all branches of physics, and few people now defy its reigning position over the formal foundation of modern science.

The theory of special relativity exerted a great impact on Heisenberg’s ideas for the development of a quantum formalism, as has been discussed in many books on the history of quantum mechanics.<sup>(5,6)</sup> Indeed, in both theories the physicists emphasized that only measurable quantities, that is, observables, belong in a theory. In quantum mechanics this has been referred to as “a guiding philosophical principle,” and in Einstein’s theory it is also regarded as “one of the keys to special relativity.”<sup>(5)</sup> However, it seems that Heisenberg did not fully realize the implications of Einstein’s relational approach to physics, which was only many years later amplified exhaustively by Bohm in his book on this subject.<sup>(1)</sup> When in 1927 Heisenberg tried to interpret the new formal structure that he had established,<sup>(7)</sup> rather than question the older ideas of physical objects in Newtonian physics, he was still, in a way, trying to retain them in the new mechanics. Indeed, such a habitual tendency to regard older modes of thought as inevitable might be quite natural, as was analyzed in detail by Bohm in the same book from a perception background, but it ultimately would also lead to ambiguity and confusion.

Thus in this paper, in analogy with Bohm’s elucidation of special relativity, we intend to interpret quantum physics as another relativistic theory. To this end the outline will be as follows. We begin in Sec. 2 with a preliminary analysis of some of the main facts underlying our use of particle and wave

pictures in quantum physics (parallel to that in Chaps. XI and XIII of Ref. 1), which are to be revealed as only consisting of relationships established in the processes of interaction. This analysis will help us not only appreciate Heisenberg’s success in formulating a quantum formalism, but also realize the inevitability of the older notions in interpreting the formalism.

There then follows a detailed discussion of Heisenberg’s interpretation which proceeds again in contrast with the discussion of Lorentz’s theory by Bohm (Chaps. IV to X of Ref. 1). Considerable emphasis is placed on its efforts to try and retain Newtonian concepts, the consequence of which is not that the interpretation disagrees with experiment, but that those concepts entering into the Heisenberg interpretation are inherently ambiguous.

After bringing out the difficulty as a result of the retention in the Heisenberg interpretation, we go on, in Sec. 3, to the adoption of a relational approach to quantum optics (similar to that of Einstein as in Chaps. XIII and XIV of Ref. 1), in which the photon is regarded as expressing a *relationship* established between a quantized light field and a photodetector in the detection. On the basis of the observed fact of the wavy modulation in the probability amplitude of the actually detected counting signals along the propagation direction, one sees that the observer cannot assign a particle trajectory for light. Thus it is clear that in the new domain of quantum investigation we need new notions for describing physical phenomena that admit the older ones, such as the particle concept, as limiting cases.

In the discussions following, we stress the roles of *field* and *interaction* as basic in the relativistic theory of light, instead of that of the *object* and its *motion*, which are basic in Newtonian theory (whereas the *event* and *process* are basic in Einstein’s special relativity, in Chaps. XXVI to XXX of Ref. 1). This leads us to an “interactive” pattern provided by the framework of quantum optics with its invariant structure that unites particle and wave pictures as two sets of relative invariant features of the same field in different frames of detection. On the basis of this unification it is made clear that the quantum failure of observers with different detection frames to acquire the same physical picture in no way falls into “subjectivism,” since the framework of quantum optics already has the perspective of the observer implicit in it.

Finally, we end this section by borrowing Bohm’s discussion of the relationship between a conceptual map and reality itself (Chap. XXXI of Ref. 1) to further remove the confusion in wave–particle duality.

The main text is then concluded in Sec. 4 and followed by an appendix on quantum paradoxes. Again, with Bohm’s discussion on the falsification of theories (Chap. XXV of Ref. 1), we show that the Einstein–Podolsky–Rosen (EPR) paradox becomes unfalsifiable within the Copenhagen interpretation, since, according to the Wigner–Araki–Yanase theorem, in the theory a great many of the physical quantities are in fact unmeasurable. In the same appendix we also summarize our resolution of the Schrödinger cat paradox.

In the whole paper, if we seem to belabor an analogy of Bohm’s exposition in his book, this is deliberate. However, if

the reader could be persuaded to believe that quantum optics is also a relativistic theory of light, then we will have achieved our aim.

## 2. OBSERVATION FOUNDATION IN QUANTUM PHYSICS

In physics the concept of a noninteracting object does not exist because its presence cannot be established. For the same reason, only by means of interaction can one discover the objective world. Consequently, *relationships established as a result of interaction exhaust all the physical facts*. To shed light on what this means to quantum theory, we begin with an analysis of some of the main facts behind our use of particle and wave models.

Two experiments that led to notions of particle and wave in atomic physics were Wilson photographs and Davisson and Germer's diffraction of matter waves. When high-energy rays pass through a cloud chamber, they cut line tracks across the vapor. From this experiment, as Heisenberg described in his book<sup>(8)</sup> *Die Physikalischen Prinzipien der Quantentheorie*, we are likely to regard the rays as consisting of minute "particles" at high speeds, with the tracks of condensed droplets indicating their trajectories. However, as Heisenberg also noticed, the formation of tracks is due to ionization when flying "particles" collide with the vapor atoms in their way, that is, as a result of interaction, by which the emerging ions then turn into original kernels causing the condensation of supersaturated vapor around them, whereupon droplets arrange themselves along the flight paths to shape tracks that are directly observed by us. Then one sees that Wilson's chamber registered only the occurrence of interactions.

From the description above one may now see that, similar to the way all the facts underlying space and time notions were analyzed by Bohm in his book,<sup>(1)</sup> the physical facts here also consist only of sets of *relationships* as a result of interaction involved in the registration (exchange of energy and momentum), in which no absolute particle is ever to be seen.

If the particle concept is only as a *relative* invariant extracted from those physical facts with certain experimental arrangement (the particle frame of detection), what then is the origin of the Newtonian idea of an absolute particle, a supposedly solid, massy, hard, and impenetrable substance, essentially independent of all relationships? "Evidently it does not come primarily from experiment and observation," as Bohm suggested,<sup>(1)</sup> but rather from the continuation in modified form of our "common sense" view of physical objects. In this view, matter is formed from discrete particles, each of which has a certain place, size, and form. Thus the particle is in effect "substantialized" and taken as an absolute.

Similarly, Davisson and Germer's detectors also recorded only the exchange processes of energy and momentum happening in the detection; wave as a notion merely represents a relative, rather than absolute, invariance of the relationships of observed facts in the corresponding circumstances (the wave frame of detection).

Through this parallel analysis to Bohm's, once the nature of

physical facts and our concepts as relative invariant features are clarified, the implications are far-reaching. First, if we recall the success of Heisenberg in 1925, we should have come to realize that it hinged essentially on considerations involving the relational properties associated with two Bohr states (spectral lines that characterize the relative energy changes), rather than any absolute property of an electron itself, tied to single Bohr orbit: *two* instead of *one*, as Dirac briefly commented.<sup>(9)</sup>

Second, if we look into Heisenberg's interpretative attempt in 1927, we will find that his validating classical concepts to interpret the quantum formalism was, fatally, an effort to retain our ordinary notions beyond their proper domain, where the theoretical frame excludes the possibility of a complete description of the particle concept (simultaneous momentum and position).<sup>(6)</sup> At this point, in order to exhibit more clearly the nature of the problems to which the older concepts gave rise in quantum mechanics, we still need to delve into the Heisenberg interpretation in some detail.

In 1900 Planck's study of the properties of radiation undoubtedly began a new page for twentieth-century physics, for it constituted the first evidence that sharply denied the basic assumption of continuity, which is essential to classical physics. It ultimately would trigger a whole revolution in our concept of physical objects. Yet it must not be expected that this should be completed in one move. Indeed, as was only natural, radical changes only occur after a long series of alternative interpretations are tried and fail, with the objective of saving our "common sense" notion of particle that is behind Newton's laws of motion. In this respect, even Heisenberg's interpretation was without exception, no matter how radical he was when he established the quantum formalism.

Heisenberg began by accepting the assumption that classical notions remain valid in quantum mechanics: "All concepts which can be used in classical theory for the description of a mechanical system can also be defined exactly for atomic processes in analogy to the classical concepts."<sup>(7)</sup> However, his basic new step was to study the dependence of the measurement of position and momentum on the relationship between the physicality of apparatus and its irreducible participation in the measurement. To do so, he constructed the famous gedanken microscope experiment to measure very accurately the position of an electron.<sup>(8)</sup> Heisenberg showed that when the indivisible quanta of action must be taken into account in the measurement process, the uncontrollable disturbance to the electron eventually made it impossible to assign simultaneously precise values of position and momentum, as regulated by an uncertainty relation. Thus in the way of considering that the apparatus was part of this physical world and must undertake the same irreducible interaction to observe, which in effect disturbed what is to be measured, Heisenberg's interpretation preserved the particle notion within the new quantum framework, that is, led to a reconciliation. (At this point let us compare this with Lorentz's way of trying to reconcile the ether hypothesis with the result of the Michelson-Morley experiment, as discussed in Bohm's book.<sup>(1)</sup> When considering that the arms of the interferometer were composed of atoms and should undergo the same shift now

called the Lorentz contraction, Lorentz actually did prove that no fringe shift could ever be detected by the apparatus of Michelson and Morley.) Nor is this all: he could even develop a whole set of uncertainty relations to imply that in quantum mechanics, because of the irreducible disturbance *all the complete descriptions of classical notions will be impossible*.

Nevertheless, the Heisenberg interpretation of the microscope experiment is formulated in terms of position and momentum of an electron, measured by an apparatus that is supposed to have an irreducible disturbance to the electron. Therefore, the measured values *ought* to be corrected to take into account the effect of the participation before we can know what they really mean. But if the Heisenberg interpretation is right, there can thus be no way to give exactly the simultaneous values of position and momentum. The *simultaneous* position and momentum that define a particle in classical dynamics are therefore inherently ambiguous, because they drop out of all observable relationships that can be found in actual measurement and experiments.

Therefore, the Heisenberg interpretation has also brought about “a novel kind”<sup>(1)</sup> of problem, which “goes to the root of basic notions that are at the foundation of physics.” Just as in the Lorentz theory on space and time,<sup>(1)</sup> the difficulty of this mainstream of the Copenhagen interpretation<sup>(10)</sup> is not its disagreement with experiment. On the contrary, it is in accord with all that has been observed since then. Rather, the problem essentially is that the *fundamental concepts* entering into the interpretation, for example, the notion of particle, are in fact *completely ambiguous*. For, as we have seen, it was deduced on the basis of Heisenberg’s uncertainty relation itself that no means at all could ever be found to give precisely to a particle simultaneous values of position and momentum. Indeed, since the complete description of classical notions of a particle cancel out of all observable results, it makes no difference whether or not we need such a classical concept of particle in quantum mechanics.

From the above discussion we have seen the remarkable similarity rooted in both the Heisenberg interpretation and Lorentz theory as detailed by Bohm.<sup>(1)</sup> Both theories were developed during a time of crisis in physics when new evidence showed certain straightforward contradiction to some basic hypotheses of classical physics (Sir Kelvin’s two clouds). To retain the older notions in the new formalism frames established by the new evidence, both theories need to refer to a mechanism of the action of apparatus in the measurement, which in effect distorts or cancels our exact knowledge of these notions. However, as a direct result, those basic notions have become intrinsically ambiguous.

According to Einstein’s relational approach to physics,<sup>(1)</sup> however, the resolution of this fundamental ambiguity involves a radical change in thinking by basing ourselves as far as possible on the facts and on hypotheses that are in principle testable. What are these facts? At the beginning of this section we analyzed one aspect of the relevant facts, viz., that all our actual knowledge of physical objects is based on observable *relationships* established by interaction. To avoid ambiguity in

our fundamental notions of physical objects, it is therefore necessary to express the whole content of physical law in terms of such relationships and not in terms of a particle with intrinsically untestable properties (e.g., simultaneous values of position and momentum) that are inherently ambiguous.

In the next section we shall show that the quantum theory of radiation, or its development since 1960s into quantum optics, provides a clear notion for the description of detection processes, which is decisive for the study of physical content in terms of those relationships.

### 3. RELATIONAL APPROACH TO THE QUANTUM THEORY OF RADIATION

Since the time of Faraday and Maxwell, physics has been developing a field theoretical description of nature. Thus our knowledge nowadays of fundamental processes is viewed through various fields and their interactions. To develop a relational approach to quantum physics, however, it is not necessary to go too far in this direction, but to concentrate our discussion on the quantum theory of light, for the reason that “in quantum optics it is often possible to address such questions from essentially first principles and to carry out accurate tests of the theory in the laboratory.”<sup>(11)</sup>

According to Maxwell’s electromagnetic theory, light is a transverse field. In a vacuum it is described by

$$\begin{aligned}\nabla \times \mathbf{E} &= -\partial \mathbf{B} / \partial t, & \nabla \times \mathbf{B} &= (1/c^2)(\partial \mathbf{E} / \partial t), \\ \nabla \cdot \mathbf{E} &= 0, & \nabla \cdot \mathbf{B} &= 0,\end{aligned}\quad (1)$$

where its total energy and momentum are

$$\begin{aligned}H &= \frac{1}{2} \varepsilon_0 \int d^3r (\mathbf{E}^2 + c^2 \mathbf{B}^2), \\ P &= \varepsilon_0 \int d^3r \mathbf{E} \times \mathbf{B}.\end{aligned}\quad (2)$$

It is appropriate to say that the rise of two of the most important principles of physics in this century, relativity and quantum mechanics, was connected to the studies of field theories. Indeed, Einstein’s special relativity was created out of the investigation of the electrodynamics of moving bodies. In contrast, quantum mechanics brought about new interpretations of the “meaning” of field theories. This began with Schrödinger, who introduced a wave equation. Based on a particle notion, Born interpreted the wave function as a probability amplitude, the square of which is the probability of finding the particle at a particular point in space. When this is applied to light, the particle is called a “photon.” This interpretation indeed is fascinating, for it can account for all phenomena that have been observed. But such a “success” is also at the cost of the key notions in the interpretation being inherently ambiguous, as discussed in the previous section.

According to this statistical interpretation, quantum mechanically, one cannot think of a classical particle as being

defined by its position and momentum, but must instead introduce *the probability of finding the particle*. In other words, the interpretation, on one hand, emphasizes explaining quantum phenomena in terms of the particle concept, but, on the other hand, it is also inferred from this interpretation itself that the completeness of description of a particle trajectory is impossible. That is to say, the particle interpretation is essentially ambiguous. This leads to much confusion. To the lay mind it seems like moving without passing through intervening space, and to the expert it seems like a “fuzzy ball.”<sup>(12)</sup> Indeed, if one cannot tell how a particle moves from one spot to the other in the space, then this particle notion would be “just purely conceptual inventions, like dotted lines that we sometimes draw in our imaginations, when we apply geometrical theorems,”<sup>(1)</sup> in order to draw conclusions concerning the real observations.

Such a problem is not just a purely theoretical one, which arises only as a result of the analysis of the Copenhagen interpretation. It is also a factual problem. For although in nonrelativistic quantum mechanics it is still possible to give a statistical interpretation over the position of a particle in the configuration space, such an interpretation can no longer exist in the relativistic frame. All the considerations from (1) the finite velocity of light,<sup>(13)</sup> (2) the impossibility of constructing a position operator,<sup>(14)</sup> and (3) the gauge invariance<sup>(13,15)</sup> indicate that the probabilistic definition of position is formally possible only in the limiting case of negligible de Broglie wavelength.<sup>1</sup>

The appearance of the quantum theory of radiation a few years later resulted in the second kind of interpretation based on field quantization, that is, the so-called “second quantization.” This scheme regards a field dynamically as a set of harmonic oscillators. Therefore, the quantization becomes a procedure to replace the pairs of normal variables into pairs of operators that have the following relations:

$$\begin{aligned} [a_i, a_j] &= [a_i^\dagger, a_j^\dagger] = 0, \\ [a_i, a_j^\dagger] &= \delta_{ij}. \end{aligned} \quad (3)$$

Then one can describe all the physical properties of a field in terms of these pairs. For example, the “global” properties

$$\begin{aligned} H &= \sum \hbar\omega_i \left[ a_i^\dagger a_i + \frac{1}{2} \right], \\ \mathbf{P} &= \sum \hbar\mathbf{k}_i a_i^\dagger a_i \end{aligned} \quad (4)$$

and the “local”

$$\begin{aligned} \mathbf{E}(\mathbf{r}t) &= \mathbf{E}^{(-)}(\mathbf{r}t) + \mathbf{E}^{(+)}(\mathbf{r}t) \\ &= i \sum \left[ \frac{\hbar\omega_i}{2\epsilon_0 L^3} \right]^{1/2} \left[ a_i \boldsymbol{\varepsilon}_i e^{i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)} - a_i^\dagger \boldsymbol{\varepsilon}_i e^{-i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)} \right] \end{aligned} \quad (5a)$$

$$\begin{aligned} \mathbf{B}(\mathbf{r}t) &= \mathbf{B}^{(-)}(\mathbf{r}t) + \mathbf{B}^{(+)}(\mathbf{r}t) \\ &= i \sum \left[ \frac{\hbar\omega_i}{2\epsilon_0 L^3} \right]^{1/2} \\ &\times \left[ a_i \mathbf{k}_i \times \boldsymbol{\varepsilon}_i e^{i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)} - a_i^\dagger \mathbf{k}_i \times \boldsymbol{\varepsilon}_i e^{-i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)} \right], \end{aligned} \quad (5b)$$

where  $\boldsymbol{\varepsilon}_i$  is a unit vector of polarization, while the quantum state of the field is represented by a vector  $\psi$  in Fock space  $|\{n_i\}\rangle$ .

Since applying  $a_i$  and  $a_i^\dagger$  to Fock states causes the states to shift, we call them photon annihilation and creation operators, respectively. It therefore suggests one may interpret the field described by  $|n_1 n_2 \dots n_i \dots\rangle$  as an ensemble of  $n_1$  particles with energy  $\hbar\omega_1$  and momentum  $\hbar\mathbf{k}_1 \dots$  and  $n_i$  particles with energy  $\hbar\omega_i$  and  $\hbar\mathbf{k}_i$  (Refs. 13 and 16). In this way, once again one obtains a particle notion but avoids the aforementioned formal difficulties. Yet it seems this time that the attempt to adjust the particle notion to observed facts has led us to a situation of more confusion in which it is no longer clear what is meant by our photon notion as a particle or what can be done with it.

In view of this deep ambiguity and confusion that has developed from the application of the intuitive notion of the particle beyond its proper domain, our approach must be, as we remarked earlier, to begin our inquiry afresh by basing it on the facts in our actual processes of light detection. Such a notion, developed by Glauber<sup>(17)</sup> in 1963, clearly describes light detection processes based on photoionization. Glauber showed that for an ideal photodetector being put at point  $\mathbf{r}$  in a radiation field, the probability of observing a photoionization, the counting signal, in the detector between  $t$  and  $t + dt$  is proportional to  $W_1(\mathbf{r}t)dt$  with

$$W_1(\mathbf{r}t) = \langle \psi | \mathbf{E}^{(-)}(\mathbf{r}t) \cdot \mathbf{E}^{(+)}(\mathbf{r}t) | \psi \rangle, \quad (6)$$

where  $\mathbf{E}^{(+)}$  and  $\mathbf{E}^{(-)}$  are the positive and negative frequency components of the electric field in (5a), and the state  $|\psi\rangle$  specifies the properties of a field at all times (the Heisenberg picture). It formulates the exact mathematical expression of our discussion on physical facts at the beginning of Sec. 2.

For simplicity, we shall consider the one-dimensional propagation of a one-photon state ( $|\psi\rangle = \sum c_i a_i^\dagger |0\rangle$ , where the  $c_i$ 's are probability amplitudes), constructed by Cohen-Tannoudji *et al.*<sup>(16)</sup> It is easy to show that the detection probability propagating along the  $x$  direction is

$$W_1(xt)dt = \frac{\hbar c}{2\epsilon_0 L^3} \left| \sum (k_i)^{1/2} c_i e^{i(k_i x - \omega_i t)} \right|^2 dt. \quad (7)$$

This probability propagation of observing photoionization *within* detectors also has reproduced the probabilistic wave of quantum phenomena that propagates without deformation with the light speed  $c$ . However, it is more essential in the physical content,

since the expression itself automatically takes into account the role of the apparatus in the detection processes.

We are now ready to adopt the relational approach to quantum physics: We shall regard *the photon as a kind of elemental "record" expressing a relationship of a light field to an actual detection process in which this record is registered*. That is, it is only a portion of energy and momentum that is transferred from a light field to a detector and by which the record is realized, and our point of departure will be the following: In terms of actually measurable "records" of this kind, the interaction between a light field and a detector in the detection processes is described by a probabilistic law as expressed by Eq. (7).

We do not regard the above result as a deduction from the Copenhagen interpretation, but as a *basic hypothesis* that is evidently subject to experimental tests and that has, in fact, already been confirmed in all the experiments until now.

To see more clearly what this hypothesis implies with regard to the meaning of the notion of particle trajectory in quantum physics, let us reconsider the Wilson-type experiment. To indicate a particle trajectory for light (or other fields), we need to arrange an array of photodetectors along the  $x$  direction to record this trajectory. However, the *fact* is that the detection processes now must follow a probabilistic law, in which the probability amplitude of counting signals in detectors along the  $x$  direction is wavily modulated as expressed by Eq. (7), since, as we have seen, experiments show this to be the case. Therefore, one can no longer draw a particle trajectory for light, because at some points the trajectory may actually discontinue.

This is a major break with Newtonian ideas, because one cannot use the notion of particle trajectory to describe quantum phenomena of light. It must be emphasized, however, that for light the establishment of the notion of particle trajectory is based only on an *indirect deduction*, the result of an organization that put together counting signals in the detectors along the  $x$  direction. Particle trajectory is therefore no longer an *immediate fact* corresponding to a light ray in our everyday experience. For it is now seen to depend, to a large extent, on a purely *conventional* procedure of assembling detection signals in the propagation direction. This convention seems natural and inevitable to our "common sense," but it leads to unambiguous results: a trajectory can be assigned only under conditions in which geometrical optics is a good approximation.<sup>(18)</sup> When the characteristic de Broglie wavelength can no longer be regarded as effectively infinitely small, then the experimental facts of physics make it clear that the absolute notion of particle trajectory should be abandoned.

It cannot be emphasized too strongly that in this relational approach one does not deduce Eq. (7) as a consequence of the disturbances of observing instruments since indivisible quanta are needed for measurement, and from this infer that a causal description is impossible in quantum physics. Rather, one begins with the experimentally well-confirmed hypothesis of the probability of interaction described by Eq. (7), as *actually measured*. This needs no explanation (e.g., in terms of the

disturbance of instruments due to indivisible quanta), but is just our basic starting point in further work. With this starting point one may expect to discover new concepts from the quantum formalism, taking the notion of particle trajectory as a *limiting case*.

Thus the new notions emerging from the framework of quantum optics are in terms of *quantum fields* (such as a light field and an electron field) and *interactions* (such as the detection of light by a photodetector). Light, as a whole, is described by a field in a quantum state  $|\psi\rangle$ , whose "global" properties are characterized by only those conservative quantities, such as energy and momentum, in the corresponding operator form of Eq. (4), acting in Fock space, while the "local" properties, such as propagation, are described by Eq. (5). However, to get any information concerning the field, an observer needs a number of photodetectors. That is, by the interaction between light and detectors one gets the immediate facts on the field. To demonstrate how physical phenomena now are described in terms of the field and interaction, we shall now focus our discussion on the propagation properties of light.

Generally, the regularity and order in the propagation properties of light can be summed up in the notion of frames of detection. This is essentially the placement of an array of detectors in a particular way, set up to make possible the expression of the results of different detections in a common language and thus to facilitate the establishment of relationships between these detections. For example, in a *particle frame of detection* one puts a series of photodetectors in the propagation direction of light. Here, the important fact is that there exists a set of *invariant parameters* among different detection processes, for example, the velocity of light signal propagation (emission and then absorption)  $c$ , which enables us not only to characterize a "trace" but also relate the "trace" to a portion of energy and momentum (a photon) transferred from light to a detector in the interaction, to form a particle picture ( $p = E/c$ ).

There also exists a *wave frame of detection*. In this frame the light is split into two paths that interfere with each other. To see the effect, one also needs to put an array of detectors on the interfering plane, from which one can infer another set of invariant parameters, such as the frequency, wavelength, and phase velocity from the interference fringes formed. Thus one constructs a *wave picture*. Indeed, as far as Newtonian mechanics is concerned, such a wave frame of detection seems unnecessary, and it makes sense to ascribe a particle notion as the only invariant feature to all the cases in the domain.

Of course, all this experience depends upon the circumstance that the de Broglie wavelength is so small that on the ordinary scale of distance and time the wave modulation in this kind of counting signal detection can be neglected. This is equivalent to assuming an infinitely small de Broglie wavelength of matter. When the finite de Broglie wavelength of matter is taken into account, as in the case in the Davisson-Germer experiment, and since light itself also behaves like a wave, new problems of "wave-particle duality" do in fact arise; these problems arose during the famous Bohr-Einstein dialog and are still a key issue

in interpreting quantum mechanics.<sup>(19,20)</sup>

In the dialog the point in dispute was the problem of physical reality, for “the dependence of what is observed upon the choice of experimental arrangement” seems so “bizarre and counterintuitive” to our common experience. However, this “observer-participancy”<sup>(19)</sup> is not peculiar only to the quantum world. As a matter of fact, it was shown in Bohm’s book with substantial scientific evidence that it is a common character to our actual mode of *perception* of the world, the implication of which is best understood from a relativistic point of view. (It would seem that this participating nature looks strange to us, mainly because of our limited and inadequate understanding of the *domain of common experience*, as Bohm suggested and discussed in detail in the appendix of Ref. 1.) Here then is our task for the following discussion.

In the procedure described above we see that the analysis of light into constituent objects (photon particles) has been replaced by its analysis in terms of quantized fields and interactions (while in Einstein’s special relativity the analysis was replaced in terms of *events* and *processes*<sup>(1)</sup>), organized, ordered, and structured to correspond to the characteristics of the light system that is being studied.<sup>2</sup> It follows that *the particle picture and the wave picture* taken jointly constitute the means by which the characteristics of physical phenomena are to be treated. In this sense particle and wave pictures together are playing a role similar to that played by the particle picture alone in Newtonian mechanics. That is to say, the nature of light is being described in terms of a kind of “interactive” pattern between a field and the detection of the observer, as exhibited in the framework of quantum optics.

In an interactive pattern, for example, of any interactive kit developed in recent multimedia culture, there is a thoroughgoing unification of its different flows of knowledge or entertainment whose courses the user can affect, based on the fact that each of the flows can be related to the others by means of some kind of directory. The question then naturally arises whether in the “interactive” pattern of particle and wave pictures taken together there is not a similar unification structure of particle and wave pictures, so that “these two aspects can be causally related with each other,” as Einstein firmly believed.<sup>(6)</sup> (Recall that in Newtonian mechanics a wave is derivatively considered as a periodic motion of particles, so that the particle concept is more basic and no such equal unification occurs there.)

To see that there is in fact such a kind of unification of particle and wave pictures in the framework of quantum optics, it is necessary only to refer to Eq. (5), in which de Broglie’s idea<sup>(21)</sup> is now expressed by the operator  $E(\mathbf{r}t)$  [ $= E^{(-)}(\mathbf{r}t) + E^{(+)}(\mathbf{r}t)$ ], in terms of the annihilation operator  $a_i$  (and creation operator  $a_i^\dagger$ ) as the amplitude with a modulating phase factor  $\exp i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$  [and its conjugate  $\exp -i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$ ]. This expression evidently contains information of propagation properties of light in the two different frames of detection. The propagation of annihilation operator  $a_i$  and creation operator  $a_i^\dagger$ , which physically describe events of absorption and emission of light, *determines* that in the particle frame of detection a light

signal travels at the speed  $c$ ; and the phase factor  $\exp i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$  in Eq. (5), due to its modulation effect in the probability expression of Eq. (6), *reflects* that in the wave frame of detection interference of counting signals of detection occurs. Thus Eq. (5) implies what one can observe in different frames of detection.

It seems clear then that in the framework of quantum optics two pictures of particle and wave are united as two sets of features of the *same* field in two different frames of detection, in which they can be related to each other in such a way that Eq. (5) is invariant. This unification can be characterized by a term called “particle-wave” rather than “particle and wave,” the dash emphasizing the new kind of unification.

It should be noted that in spite of the above-described unification of particle and wave brought about in the framework of quantum optics, there remains a rather important and peculiar distinction between them, resulting from the fact that  $a_i$  and  $a_i^\dagger$  are operators, but the phase factors  $\exp i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$  [ $\exp -i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$ ] are c-numbers. On the basis of this distinction, it is also made clear that the modulation wave in the probability amplitude of counting signals at a velocity (phase velocity) greater than  $c$  in de Broglie matter systems in no way confuses us on the maximum speed of propagation of signals, provided that a signal propagation is described by the annihilation and creation operators  $a_i$  and  $a_i^\dagger$ .

The implication of the framework of quantum optics can be made clearer, by which much of our confusion in the wave-particle duality can be avoided, if one still follows Bohm’s discussion of the Minkowski diagram to explain it as a kind of conceptual map. As we know in special relativity, the Minkowski diagram also serves as an invariant structure by which one can relate the measurements of space and time coordinates in different *frames of reference* to the same *event*.<sup>(1)</sup>

Because of the relativistic unification of particle and wave pictures into a single expression, Eq. (5), there appears an illusion of coexistence of wave and particle pictures. However, a little reflection shows that this view of the framework of quantum optics must be very far from the truth indeed. Consider, for example, that if an observer wants to measure the speed of a light signal, he must construct a particle frame of detection that registers both where and when a light signal is emitted and then absorbed. (The propagation of a light signal is in fact a subject of special relativity.) Such an observer cannot survey the whole of Eq. (5). On the contrary, he can only know of the propagation of annihilation and creation operators  $a_i$  and  $a_i^\dagger$ . Therefore, the exact information of the phase factor  $\exp i(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$  is unknown to him; that needs an interference experiment.

The real situation, as experienced by an observer at one of the frames of detection, is indeed strikingly different from what is shown in Eq. (5). An observer’s knowledge is restricted to the part of Eq. (5) (e.g., the amplitude part  $a_i$  and  $a_i^\dagger$ ) that is in the particle frame, and he never sees what is found of the other part (the phase factor) in the wave frame, *as it is represented in Eq. (5)*. For in any frame of detection we are experiencing only

what is actually present in that frame. What we see in the wave frame no longer actually exists in the particle frame. What is left of the wave experiment done before is only a record of detection. This record may be in our memories or on a photographic plate. From these records we *reconstruct* a wave picture in our thoughts as well as with the aid of pictures and models.

Of course, as Bohm also conclusively illustrated with the example of the Minkowski diagram,<sup>(1)</sup> our notions of physical phenomena are in fact all based on a reconstruction, “in accord with appropriate geometrical, dynamical, structural principles that have been abstracted from a wide range of past experiences.” In this sense, the framework of quantum optics will also be a kind of conceptual map, having a structure that is similar to that of real sets of light fields and interactions that can actually be observed. “Any map of this kind *is* what the world *is not*.” That is, the framework of quantum optics consists of operators and states of an operator calculus in Hilbert space, while the experiments in the real world contain laser sources, beam splitters, photodetectors, and so on. But as happens with the framework, it implies a structure similar to the structure of *what is*.

“In all maps (conceptual or otherwise) there arises the need for the user to locate and orient himself by seeing which point on the map represents *his* position and which line represents the direction in which *he is* looking.” In doing this, one recognizes that every act of “actualization”<sup>(20)</sup> as in the discussion of wave-particle duality yields a unique perspective on the world. But with the aid of the framework of quantum optics, one can relate what is seen from one perspective (the particle frame) to what is seen from another (the wave frame), in this way abstracting out what is invariant under change of perspective and leading to an ever-improving knowledge and understanding of the actual character of the radiation under investigation. Thus if an observer, conducting experiments with different frames of detection, is to understand what he sees, he need not puzzle over which view is “right” and which view is “wrong” (wave or particle). Rather, he consults the map, Eq. (5), and tries to reach a common understanding of why each way detecting the same light field has a different perspective and comes therefore to his one view, related in a certain way to that of the other (e.g., the de Broglie relation  $p = h/\lambda$ ).

In this way, we unite two pictures of wave and particle as two sets of invariant features of the same light field in different frames of detection. The notion of particle in Newtonian mechanics now is an approximation under the circumstance that the effective de Broglie wavelength is infinitely small, whereas in another limiting case when the average photon number is large enough so that discrete phenomena of quantization are washed out, we recover the concept of electromagnetic wave of Maxwell’s theory.

Of course, the story of quantum relativity does not stop here. In the “single counting signal”<sup>(16)</sup> domain of detection discussed above the notion of particle can still be contained as an approximation, which enables Heisenberg to account for quantum phenomena by means of a disturbance. But in the

double-counting signal (second-order) domain, “Heisenberg’s microscope experiment breaks down.”<sup>(10)</sup> The Newtonian notion of particle can no longer explain long-distance correlation phenomena without violating special relativity, because there must be a nonlocal informing mechanics between two separated particles. (This was also realized by Bohm.<sup>(22,23)</sup>) These phenomena, however, can be explained as the correlation of *local* interactions of a *global* quantized field in a state  $|\psi\rangle$ . Thus when we come to this new domain of experience, it is not surprising that new concepts are needed, leading to an understanding of the new phenomena under investigation.<sup>(23)</sup>

Milonni has shown that this kind of phenomena can be unitedly described in terms of the second-order correlation function  $W_{\Pi}(rt, r't')$  in the detection theory.<sup>(23,24)</sup> The probability of double-counting signals that a photoionization occurs at  $r$  between  $t$  and  $t + dt$  and another one at  $r'$  between  $t'$  and  $t' + dt'$  is proportional to  $W_{\Pi}(rt, r't')dt dt'$ , where

$$W_{\Pi}(rt, r't') = \sum_{m,n} \langle \psi | \mathbf{E}_m^{(-)}(rt) \cdot \mathbf{E}_n^{(-)}(r't') \cdot \mathbf{E}_n^{(+)}(r't') \cdot \mathbf{E}_m^{(+)}(rt) | \psi \rangle \quad (8)$$

with  $m, n$  summing over polarization axes  $x, y$ , and  $z$ .

This joint counting probability of Eq. (8) by its nature suggests an invariant relationship in this domain. If we take the coincidence of two photoionization events (the complete correlation) as the manifestation of the *wave feature*, whereas the *particle feature* means no joint counts ever occur, then for a light field in state  $|\psi\rangle$ , we have the following identity:

$$dP_{wf} + dP_{pf} = 1, \quad (9)$$

where the differential probability  $dP_{wf} = W_{\Pi}(rt, r't')dt dt'$ , and by definition the probability of no joint counts  $dP_{pf} = 1 - W_{\Pi}(rt, r't')dt dt'$ . Therefore, in this way we also unify the wave feature and the particle feature into a continuum within this second-order domain. Thus we see that not only the way the quantized field is detected but also how it is generated (in different quantum state  $|\psi\rangle$ ) play important roles in the physical laws in the quantum theory of radiation.

Finally, we shall follow Bohm<sup>(1)</sup> to reach the following summary. In Newtonian mechanics the role of the observer was very much underemphasized; physicists may have thought that the perspective of the observer need not appear in the fundamental laws of physics, although they may have always learned that each observer does have a perspective. Rather, they described physical phenomena in terms of motion of “absolute” particles that are independent of the way in which they are measured and observed, so that no part is played by the observer at all in these laws.

On the other hand, according to the relational approach to physics, it is clear that the framework of quantum optics is a map corresponding to what will be observed in a frame of detection arranged in a certain way. Therefore, this map has already taken into account some of the observer’s perspective. Moreover, as we have seen, not only the way of detecting a field

but also the way of preparing it (in a state  $|\psi\rangle$ ) has a different perspective to the field in the second-order detection domain. Thus, whether or not we consider what is seen by different observers or by the same observer in different frames, it is always necessary to relate the results of these detections by referring to a particle-wave map with a correct structure “and in this way develop an ever-growing knowledge and understanding of what is invariant and therefore not dependent on the special perspective of each observer.”

#### 4. FURTHER DISCUSSION AND CONCLUSION

The development of modern physics has shown its striking tendency to get away from “absolute” notions. Newtonian mechanics, as the first main theory of physics of this kind, had already incorporated a number of relativistic ideas that underlie our use of the Galilean transformation. But in the theory the three basic notions of space, time, and physical objects were still treated as absolute.

The radical revolution in our concepts of space and time initiated by Einstein in some way depends upon how to understand a new transformation (the Lorentz transformation) discovered in electrodynamics.<sup>(5,25,26)</sup> In terms of the *old notions* of Newtonian physics, that is, the “real” (or “true”) time, Lorentz thought that the time that entered into the transformation relations was the “apparent” time. But the “very famous point of Einstein” was essentially to base facts and hypotheses that are in principle testable. “There is not one ‘apparent’ and another ‘real’ time; there is just one ‘real’ time, and that is what Lorentz called ‘apparent’ time.”<sup>(25)</sup> In terms of this *real* time that consists only of a relationship between the observed phenomenon and apparatus, new concepts concerning space and time then are necessary.

The fundamental changes in our notion of physical objects, however, reside in the quantum formalism, especially in the fact-oriented framework of quantum optics. Such a new formalism might also be expected to lead us to new concepts, which would contain the older ones as approximations and limiting cases. With Einstein’s relational approach to physics, in this paper it is proposed that the *real* photon, entering the new framework of quantum optics, also expresses only an elemental relationship between a light field and a photodetector that we can really observe in detection, and it is hoped that this treatment will bring about a “turning around of the physical picture.”

In Einstein’s special relativity the roles of the *event* (such as emission or absorption of signal) and *process* (such as the transmission of a signal) were introduced to replace those of *object* and its *motion*, which are basic in Newtonian theory, where the Minkowski diagram serves as a conceptual map that already has the observer’s perspective implicit in it. In the quantum relativity discussed here, however, an analysis in terms of the *quantized field* and *interaction* is further suggested to account for those events of emission and absorption in the domain of quantum phenomena, in which the framework of quantum optics, the same, becomes a map that can tell us what are to be observed in different frames of detection.

It can be said that in Einstein’s relational approach, the whole

task of physics is assumed to find out what is relatively invariant in the study of relationships between various aspects of this universe. Such a guiding epistemology should also be of utmost importance to quantum physics, since it seems that conceptual difficulties arise whenever we refer to particles and waves as more or less permanent objects, rather than regard them as relative invariants that have been abstracted from a variety of relationships of observation; and it seems that once we can decompose the problems into fundamental processes in terms of the interaction between a light field and detectors, one would in principle access the key to the problems.

In terms of the field and interaction, basic change in our notions of quantum measurement is to be expected, in which one no longer regards the interaction as a disturbance factor from an observing apparatus to an object and from this infer that all the complete descriptions of the object are impossible as specified by Heisenberg uncertainty relations. Rather, one should *utilize* the interaction as a means to build up relationships between the observing apparatus and the observed universe to find out the invariant structure of the universe.

In terms of the field and interaction, the probabilistic “wave” of quantum phenomena in Eq. (7) represents only the probability of *interaction event* (emission or absorption of a portion of energy and momentum) happening in the processes of detection. We hope that this kind of probabilistic hypothesis would have pleased Einstein, for he opposed only the *ontological* probability.

In terms of the field and interaction, problems such as interference between independent laser beams will no longer puzzle us by posing questions such as, How does a photon interfere? With itself<sup>(27)</sup> or with others?<sup>(28)</sup> Since Eq. (6) describes the probability of an interaction event in which a portion of energy and momentum (a photon) is transferred from one of the light fields to a detector, the detector certainly does not discriminate from which field it received the photon, and the phenomenon can be well explained by the interference of transition amplitudes<sup>(16)</sup> in the framework of quantum optics.

In terms of the field and interaction, not only wave-particle duality discussed above, but also the other kinds of duality revealed recently<sup>(29)</sup> can be understood by taking the framework of quantum optics as a map that has implied the role of the observer. Thus the “potentiality under the actualization”<sup>(20)</sup> should be understood as a *potentiality of interaction* in which the observer can choose *freely* his detection frame, rather than an *ontological* potentiality by which one thereby falls into a kind of “subjectivism.” The measurement of polarization discussed in Ref. 20 is just such an example; it not only illustrates the principle of superposition of states in quantum mechanics,<sup>(27)</sup> but also plays a great role in Bohm’s EPR-type experiment.<sup>(22)</sup> The main facts establishing the superposition law, however, are still based on interaction. Every light field does have its definite (rather than “ambiguous”) polarization direction. But to measure the polarization one must employ a polarizer, the function of which is to “project” an unknown but definite mode onto two known perpendicular directions; that is, by absorbing a photon of the mode and then emitting one of the two modes with probability, we can then assign the polarization. As we know, in

the experiment the probability depends upon the orientation of the polarizer that we can freely choose. Thus we see that quantum theory does emphasize the special role of each observer in a way that is different from what is done in Newtonian mechanics. "But the recognition of this unique perspective serves, as it were, to clear the ground for a more realistic approach to finding out what is actually invariant and not dependent on the perspective of the observer."<sup>(1)</sup>

To arrive at the final conclusion of this paper, let us recall *a priori* assumptions of Newton in the Introduction. After we have in effect followed Bohm's relativistic melody to hum a quantum song, it will be realized that it is our mankind selves who "in the beginning form'd matter in solid, massy, hard, impenetrable, moveable, ..., as most conduced to" by our everyday life in the man-sized domain. Such a notion generally is adequate only in this domain of validity, so that as we go beyond this domain, we may expect to come to the development of new concepts. In the progress of this process, twin ideals of Galileo's scientific methods, experimentation and mathematization, along with Einstein's relational approach to physics, will forever have won.

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#### APPENDIX: QUANTUM PARADOXES — THE EPR PARADOX AND SCHRÖDINGER'S CAT

After we have adopted the fact-oriented framework of quantum optics to bring out new notions for describing quantum phenomena, in this Appendix we return to the land of quantum mechanics where fundamental problems originally arose.

One of the problems of the Copenhagen interpretation that concerned Wigner very much is that of "unmeasurable quantities."<sup>(30)</sup> Early in 1952<sup>(31)</sup> Wigner showed that quantities that do not commute with all additive conserved quantities cannot be precisely measured. Araki and Yanase<sup>(32)</sup> later proved the theorem mathematically for the general case, and a detailed discussion of the physical implications has been provided in Wigner's Princeton Lectures.<sup>(30)</sup> The Wigner-Araki-Yanase (WAY) theorem has in fact posed the severest problems to the "standard interpretation," not only because the results "blur the mathematical elegance of von Neumann's original postulate that all self-adjoint operators are measurable," but also because, if those quantities, in a strict way, are unmeasurable, *it makes no difference whether or not we assume that there are such quantities*, according to Bohm's opinion, which was intensively discussed in his book.<sup>(1)</sup>

The "difficulties inherent in the measurement of a great many, if not most, operators" were shown to be the "internal" problems of the standard interpretation. Therefore, if those quantities that von Neumann called also observables are in fact unobservables, then one should not interpret quantum mechanics in terms of those notions, since in quantum theory the "very famous point of Einstein" was also emphasized that only measurable quantities belong in a theory<sup>(25)</sup> to avoid making

unnecessary and unprovable assumptions concerning those quantities that are unmeasurable.

Henceforth position by itself and other quantities of this kind by themselves that do not commute with conserved quantities "are doomed to fade away into mere shadows"<sup>(26)</sup> (as bare operators in quantum formalism), and *only those conservative quantities will preserve the physical reality*.

Thus position, whose physical reality must be unequivocally determined by the structure of quantum mechanics, is "first deposed from its high seat," and new concepts are needed as discussed in the main text.

In view of the "very famous point of Einstein," the WAY theorem that *only quantities that commute with all additive conserved quantities are precisely measurable* can also offer us new sights into EPR paradox.<sup>(33)</sup> In 1935 Einstein, Podolsky, and Rosen studied a system in which two "particles" interact with each other first and then the interaction ceases (it is so arranged that the measurement of one of the two "particles" can be performed without in any way disturbing it). According to the "standard interpretation" that every observable quantity corresponds to a self-adjoint operator, they reached this conclusion: (1) when the operators corresponding to two physical quantities do not commute, the two quantities do have simultaneous reality. This obviously contradicts the property of self-adjoint operators in Hilbert space: (2) when operators corresponding to physical quantities do not commute, the two quantities cannot be simultaneously measured, that is, a paradox. Quantum mechanics was so successful in explaining phenomena in the atom-sized world that EPR did not question what Bohm later called "the inherent ambiguity" in the von Neumann axiomatic system, but instead doubted the completeness of quantum mechanics.

Now, in accordance with the WAY theorem, we see that the two quantities (position and momentum) that EPR assigned, the measurability of position whose corresponding operator does not commute with Hamiltonian, are actually completely ambiguous. Thus it is a sure thing that the paradox, due to the intrinsic vagueness of the measurability of "a great many" quantities, will be forever "unfalsifiable" *within* the standard interpretation, a term that Sir Popper used to describe any such kind of proposition. (For more details please see Chap. XXV of Ref. 1.)

Besides the ambiguity of measurability for many important quantities that formed the Copenhagen interpretation, there are other fundamental weaknesses of the standard theory. In his Princeton Lectures Wigner also reformulated resolutions for those problems related to the measurement paradox. To bring out the point directly, let us begin with the discussion of quantum jumps.<sup>(34)</sup> In quantum mechanics such a process of quantum jumps is described by a system with coupling (i.e., in interaction with the external surroundings).

From substantial experimental spectral facts it is firmly verified that some of the physical elements of atomic systems (e.g., energy) only take discrete values. Hence the development of those elements are in a jumplike style, that is, quantum jumps. However, for a system with coupling the evolution of the Schrödinger wave function with time is continuous. Thus it is

obvious that *the development of physical elements (physical quantities) and the evolution of the wave function of a quantum system are by no means the same process.*

In classical physics the continuity is assumed to be a basic feature of physical systems. That is to say, by interaction energy and momentum are transferred continuously from one system to another. However, Planck's quantum hypothesis thoroughly altered this picture to give quantum jumps between discrete eigenvalues of atomic systems. It is also this "all-or-nothing" nature<sup>(23)</sup> of Planck's quanta that gives rise to the intrinsic chance of quantum events of interaction, in which one can only plead to the *objective probability*,<sup>(20)</sup> as we have discussed above (probability out of the "or" relation).

Thus it is evident that there need to be two different mathematical entities in the quantum formalism in order to describe completely quantum phenomena: eigenvalues, which are related with eigenfunctions determined from eigenequations of operators in a system, represent physical elements of reality; the Schrödinger wave function, which is the solution of a time-dependent Schrödinger state equation, gives the probability of quantum jumps between those eigenvalues. (A Schrödinger wave function, in general, is expanded into a series of eigenfunctions, where the coefficients are explained as the probability amplitudes of finding the eigenvalue corresponding to that eigenfunction in the system.)

To appreciate the necessity of the double-track description, we shall discuss an example of a two-level atomic system decaying in an electromagnetic background.

If the system is in interaction with a free space, the effects of the coupling on the two processes are as follows: (1) for the Schrödinger wave function,

$$|\psi(t)\rangle = c_a(t)|\psi_a\rangle + c_b(t)|\psi_b\rangle, \quad (\text{A1})$$

where  $|\psi_a\rangle$  and  $|\psi_b\rangle$  are the eigenfunctions corresponding to the eigenvalues of upper energy  $E_a$  and lower energy  $E_b$ . In the Weisskopf-Wigner approximation the probability  $c_b(t)$  that the system can potentially take the eigenvalue  $E_b$  is growing with time exponentially. It is seen that the Schrödinger wave function  $|\psi(t)\rangle$  evolves continuously and causally.

(2) For the physical elements the coupling to the free space provides a possible way out for the developing. Since whenever a jump occurs at later times, the quantum emitted to the electromagnetic field will propagate away in the open space making the transfer irreversible. Thus, if one follows the behavior of the atom, it will stay all along in the upper level until a jump occurs, in which the time of jump is distributed with probability determined by  $c_a(t)$  [or  $c_b(t)$ ].

If the system is put in a closed space (e.g., an atom in a microcavity), the effect of the coupling will lead to the following: (a) the above evolution of the Schrödinger wave function is modified and quantum recurrence of Rabi oscillation occurs; (b) since the reflection of the electromagnetic field by the cavity walls, the jump in the development of physical elements is effectively prevented,<sup>(35)</sup> and the system will always stay in the upper level, that is, no-go, no matter the oscillation

in the evolution of the Schrödinger wave function. Nevertheless, when the atom is flying across the microcavity at a certain speed, the whole system of atom and cavity should be taken as a "caviton," which is also in interaction with the free space. This "caviton" can emit an atom into open space that will fly away. Therefore, at the moment when the atom leaves the cavity, as in the first case, the probability of emitting the atom in the upper state is still determined by the evolution of the Schrödinger wave function, that is, by the interval time of the atom flying across the cavity, as was verified by experiment.<sup>(36)</sup>

Certainly these two dynamics interplay with each other. For instance, in a simple case of decaying in a multilevel system once a jump occurs in the developing process of physical elements, the evolution of the Schrödinger wave function will start from new initial conditions. And in more complicated systems there is the effect of "back action of measurement," which has now been well studied in quantum optics to include phenomena such as quantum jumps, dynamics of micromaser, and continuous photodetection.<sup>(37)</sup> In these cases a jump in the developing process of physical elements will drastically affect the way that the Schrödinger wave function evolves, and as a repay, the evolution determines the probability of the next jump.

With the above clarification that eigenvalues of operators describe the physical elements of reality, whereas the Schrödinger wave function will give the probability of quantum jumps between these eigenvalues, it seems clear that if one attaches the Schrödinger wave function to the physical element, then it "is consistent with modern quantum mechanics only if the temporal evolution of the system is such that a coherent superposition of the states does not develop."<sup>(34)</sup> Instead, if one pins the physical element on the Schrödinger wave function, then the physical element will be "blurred,"<sup>(38)</sup> because it can take different values at the same moment. Thus it would be "naive" to accept a "blurred model" for representing reality, according to Schrödinger.

We have seen that it was the absurdity of a "blurred model" of the Copenhagen interpretation that upset Schrödinger to propose "the case of cat." However, there will be no paradox here if one examines his executing device in detail. The paradox was caused by our ignorance of a very simple fact that *two matter systems never interact directly, but are mediated by an electromagnetic field.*

The derivation of the cat paradox is based on a coherent evolution of the state of the total system that leads to the entanglement between the state of the observed system and that of apparatus.<sup>(6)</sup> However, in all the cases of this kind of decaying problems, as analyzed above, the coherent evolution is between an atom and electromagnetic background, that is,<sup>(39)</sup>

$$|\psi(t)\rangle = c_{a,\{0\}}(t)|\psi_a\rangle|\{0\}\rangle + \sum_r c_{b,\{1\}r}(t)|\psi_b\rangle|\{1_r\}\rangle, \quad (\text{A2})$$

rather than between the atom and a detector, the cat, where multimode  $\{0\}$  expresses the vacuum state of radiation and  $\{1_r\}$  the state of one photon in the  $r$ th mode and none in the other;  $c_{a,\{0\}}$  and  $c_{b,\{1\}r}$  are probability amplitudes. Thus it is clear that

there is neither a waver between the “cat alive” and the “cat dead,” nor even the states that are assigned to them in the Copenhagen interpretation. The “alive” or “dead” of an innocent cat is only an indication of whether a quantum jump occurs in the developing process of physical elements of the atom, as discussed above, to which the coherent evolution of the state of *an atom and the electromagnetic background* in Eq. (A2) corresponds. In other words, long before the box is opened, it has already been determined that the cat is alive or dead. There

fore, there is nothing paradoxical in the state of affairs.<sup>(20)</sup>

Although the Schrödinger cat might no longer bewilder us, the study of so-called Schrödinger cat states in quantum optics out of the “cat affair” is important to the understanding of the quantum statistical properties of radiation. It will be useful in the state engineering of radiation fields and may find its applications, for example, in coherent chemistry.<sup>(40)</sup>

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### Résumé

*Par analogue avec l'explication de Bohm de la relativité restreinte, ce papier consiste en une critique de l'interprétation de Copenhague et la recherche d'une manière de réinterpréter la théorie quantique à la lumière de nouvelles observations en optique quantique. En cela, nous sommes amenés à considérer le photon réel comme exprimant seulement une relation élémentaire établie entre un champ lumineux quantique et un détecteur. Une telle approche de la physique quantique exige de nouveaux concepts pour décrire les phénomènes physiques. La structure mathématique de la théorie quantique des radiations est alors vue comme un plan conceptuel de la même manière que le diagramme de Minkowski pour la relativité restreinte d'Einstein, qui comprend déjà implicitement la perspective de l'observateur.*

### Endnotes

- <sup>1</sup> We shall reconsider the problem from a different angle in the Appendix.
- <sup>2</sup> Of course, in the earlier quantum theory of radiation, the physicists probably always realized the notion of field. However, they may have felt that such a concept of field need play no part in the interpretation of physical phenomena. Rather, they assumed more or less that the physical objects are described in terms of an ensemble of photon particles.

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